Adaptive Finite Differences Method and Parameter Selection for Total Variation Minimization

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The Model

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We propose the following model to denoise g :

Model (denoising)

$$\inf_{u \in \mathrm{BV}(\Omega)} \frac{1}{2} \int_\Omega \alpha(x)(u-g)^2 \, dx + \int_\Omega \lambda(x)|Du|,$$

where,

- $\mathrm{BV}(\Omega)$ is the space of functions with bounded variations,
- Du is the total variation (measure),
- $\alpha, \lambda : \Omega \to \mathbb{R}_+$ are parameters.

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- We want to regularize more on the homogeneous parts and to be close to the data on the edges of the image.

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Why AFDM instead of FEM?

- Mesh adaptivity : error indicator

$$\eta_h := (u_h - g)^2,$$

and a *bulk criterion* (windowing technique + statistical argument) to determine the presence of noise or of edges.

Automatic Parameters Selection

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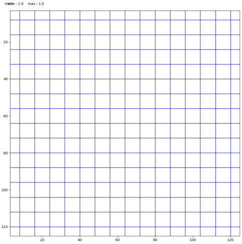
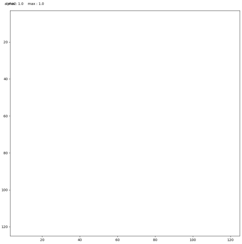
- λ adaptivity : with AFDM, the value of the discrete TV is mesh-dependent i.e. the balance between the data-fitting term and the TV change over the iteration. We compensate this change by changing λ.

(a) Original image f

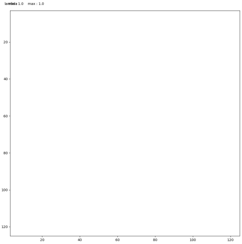


(b) Mesh

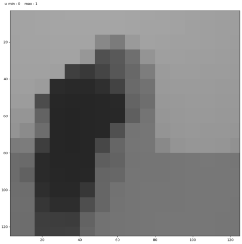


(c) α

(d) Noisy image g

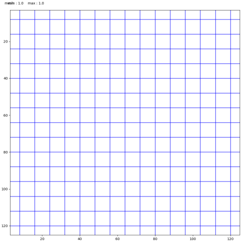


(e) λ

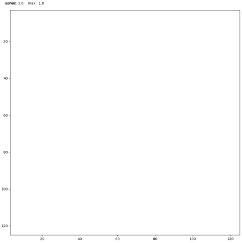


(f) Reconstruction u

(a) Original image f



(b) Mesh



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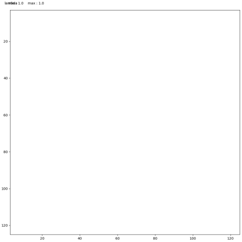
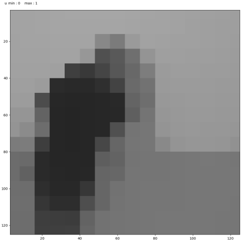


(e) λ



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Future research :

- add a L^1 data-fitting term to deals with impulse noise,
- add coarsening of the mesh,
- have elements smaller than 1 pixel to enforce the discontinuities,
- zooming, deblurring, computing optical flow.

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Thanks for your attention!